

Curriculum Map for: AP Calculus BC

Prepared June 1, 2007 by Lynn H. Greene revised August 23, 2007

Textbook: Finney, Demana, Waits & Kennedy *Calculus: Graphical, Numerical, Algebraic* Prentice Hall 2007 3rd edition

Instructional Materials: Textbook, solution manual for each student, PC for computer software demonstrations, TI-83 Plus graphing calculator, ELMO Visual Presenter.

Prerequisites: Before studying calculus, all students should complete four years of high school mathematics designed for college-bound students: courses in which they study algebra, geometry, trigonometry, analytic geometry, and elementary functions. These functions include those that are linear, polynomial, rational, exponential, logarithmic, trigonometric, inverse trigonometric, and piecewise defined. In particular, before studying calculus, students must be familiar with the properties of functions, the algebra of functions, and the graphs of functions. Students must also understand the language of functions (domain and range, odd and even, periodic, symmetry, zeros, intercepts, and so on) and know the values of the trigonometric functions at common angles and angles co-terminal with the common angles.

Scope AP Calculus BC consists of a full high school academic year of work that is comparable to calculus courses in colleges and universities. It is expected that students who take AP Calculus BC will seek college credit, college placement, or both from institutions of higher learning. Success in AP Calculus BC is closely linked to the preparation students have had in courses leading up to this course. Students should have demonstrated mastery of prior material. AP Calculus BC is primarily concerned with developing the students' understanding of the concepts of calculus and providing exposure to its methods and applications. *The course emphasizes a multi-representational approach to calculus, with concepts, results, and problems being expressed graphically, numerically, analytically, and verbally.*

Assessment:

Assessment comes in a variety of forms and wherever possible is designed to reflect and enhance the teaching and learning process that occurs in a classroom. Assessment is not a separate activity, but as an integral part of the teaching and learning process. Alternative assessments apply to any and all assessments that differ from multiple choice, timed, one-shot approaches that characterize most standardized and classroom assessment. Authentic assessments are assessments that engage students in applying knowledge and skills in the same way they are used in the real world. Performance assessment is a broad term, encompassing many of the characteristics of both authentic and alternative assessments.

All assessments in AP Calculus BC are modeled after the AP exam. Sections include multiple choice and free response. In the free response sections of assessments, students are expected to show their methods and explain their thought processes.

The **TIME** column offers a suggested time-line so that all topics listed in the **CONTENT/SKILLS** column are feasibly met. It is understood that times will need adjustments as the course develops. The **APPLICATION/PROJECT IDEAS** column offers suggestions and sources for the teacher. This column should be updated periodically to keep current and as new ideas are generated. The **KEY IDEA/PERFORMANCE INDICATOR** column coordinates topics with the New York State standards.

TIME	CONTENT/SKILLS	APPLICATIONS/PROJECT IDEAS	KEY IDEA/PERFORM INDICATOR
September (9 Days)	<p><u>Topics from Precalculus</u></p> <ul style="list-style-type: none"> • Produce the graph of a function on a coordinate system with the aid of graphing calculator. • Identify the zeros of a function. • Explore local and global behavior of a function. • Analyze functions graphically, numerically, and analytically. • Analyze rational inequalities. • Use the circle equation and explore tangent lines to the circle. • Review trigonometric functions and values. 	<p>An intensive trigonometry review is helpful here. Students will benefit from a review packet focused on progressing them through trigonometric concepts like co-terminal angles and analyzing graphs.</p> <p>Students spend time in this first unit working in cooperative groups to begin to learn how to use algebra in conjunction with the graphing calculator to analyze functions; including looking for zeros, relative and absolute maximums and minimums as well as the “pitfalls” of calculator analysis alone.</p> <p>This unit also allows students to begin to use rigorous mathematical vocabulary in homework and in oral class and small group presentations.</p>	<p>ALL TOPICS ARE FROM THE COLLEGE BOARD. NEW YORK STATE FOLLOWS THE COLLEGE BOARD CURRICULUM FOR THIS COURSE.</p>
September (7 Days)	<p><u>Limits and Continuity</u></p> <ul style="list-style-type: none"> • Calculate limits using algebra. • Estimate limits using graphs and tables of data. • Describe asymptotic and unbounded behavior in terms of limits. • Interpret continuity in terms of limits. • Understand and perform the Continuity Test. 	<p>It is helpful to give students an exploration activity with the graphing calculator where students investigate the behavior of 10-12 functions. After the investigation, students categorize the functions to form general rules about limits.</p> <p>As a final synopsis, students from small groups present the group’s conclusions to the class.</p>	

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October (10 Days)	<p><u>Differentiation</u></p> <ul style="list-style-type: none"> • Define the derivative as the limit of the difference quotient. • Analyze derivatives graphically, numerically, and analytically. • Explain the relationship between differentiability and continuity. • Interpret the derivative as the instantaneous rate of change of a function. • Apply the Mean Value Theorem • Relate the tangent line to a curve and local linear approximation to the derivative. • Approximate the rate of change from graphs and tables of values. • Calculate first and second derivatives. • Verify and generate the graphs of functions using the graphs of f' and f''. 	<p>Starting derivatives with some basic review of average rate of change and slope is helpful.</p> <p><i>Here students spend time working together in small groups to develop strategies for analyzing derivatives; including comparing and contrasting the efficiency and accuracy of using algebraic skills, the numeric derivative keys on the graphing calculator, and the graphing and table features on the graphing calculator.</i></p> <p><i>At this point in the course we begin to discuss and apply the concept of “justify your answer” on the AP Exam.</i></p>	
Oct.-Nov. (15 Days)	<p><u>Applications of Derivatives</u></p> <ul style="list-style-type: none"> • Analyze curves using the notions of concavity. • Utilize differentiation rules for basic functions including exponential, logarithmic, and trigonometric and inverse functions. 	<p>Spend 2-3 days specifically on sketching graphs from information gathered from the first and second derivatives. Students often struggle with this. Charts with characteristics are helpful.</p>	

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	<ul style="list-style-type: none"> • Apply basic differentiation rules for sums, products, quotients. • Utilize the chain rule and implicit differentiation. • Model rates of change and related rates in real-world problems. • Solve optimization problems examining both absolute and relative extrema. • Interpret the derivative as a rate of change in varied applied contexts, including velocity, speed, and acceleration. • Model rates of change, including related rates problems. • Interpret differential equations geometrically via slope fields. • Describe the relationship between slope fields and solution curves for differential equations. • Estimate numerical solution of differential equations using Euler's method. • Analyze and compute limits using L'Hospital's Rule and recognize indeterminate forms of expressions. 	<p>In this chapter students are introduced to the Free Response questions of the AP Exam. We discuss the difference between “calculator active” and “calculator inactive” questions. Students first look at models of good solutions and then develop their own models; first, in small groups and then individually.</p> <p>From this point on in the course students are required to complete, and turn in for a grade, two free responses each week. In addition, students take turns presenting their solutions to the class in order to practice oral presentations designed to develop rigorous use the vocabulary of the course as well as to practice developing correct and complete solutions.</p>	

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November (9 Days)	<p><u>Integration</u></p> <ul style="list-style-type: none"> • Estimate the area under a curve using geometric formulas. • Compute Riemann sums using left, right, and midpoint evaluation points. • Define the definite integral as a limit of Riemann sums over equal subdivisions. • Identify the integral as the accumulator of the quantity of the rate of change function. • Use basic properties of integrals. • Evaluate integrals on the graphing calculator using both the numeric and graphing features. 		
Nov. - Jan. (35 Days)	<p><u>Applications of Integration</u></p> <ul style="list-style-type: none"> • Recognize that the techniques of antidifferentiation follow directly from derivatives of basic functions. • Determine antiderivatives by substitution of variables including change of limits for definite integrals. • Evaluate definite and indefinite integrals using integration by parts. • Evaluate antiderivatives by substitution of variables including change of limits for definite integrals. • Evaluate antiderivatives using the technique of simple partial fractions using nonrepeating linear factors only. 	<p>After being instructed on integration, send students to websites that demonstrate various techniques for finding volume. Have them critique the websites for clarity and content.</p> <p>Given a variety of types of definite integrals, students “race” ,in teams of 2-3, to compare which strategies are fastest (graphing calculator using the numeric key, graphing calculator using the graphing keys, algebraic). At the conclusion of the event, students verbally explain when each strategy may be the most efficient to solve a problem.</p>	

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Jan.-Feb. (8 days)	<ul style="list-style-type: none"> • Investigate physical, social, and economic situations with appropriate integrals. • Determine the area and volumes of geometric figures using integrals. • Find specific antiderivatives using initial conditions. • Use separable differential equations to study the equation $y' = ky$. • Solve logistic differential equations and use them in modeling examples. • Find the volume of a solid with known cross sections. • Use the Average Value Theorem to approximate integrals. • Approximate definite integrals of functions represented algebraically, graphically, and by tables of values using the Trapezoid Rule and Riemann Sums. <p><u>Polar Functions</u></p> <ul style="list-style-type: none"> • Convert between polar and Cartesian coordinates. • Rewrite functions using polar and Cartesian form, emphasizing the three relations $x = \cos\theta$, $y = \sin\theta$ and $x^2 + y^2 = 1$ • Analyze planar curves given in polar form, including velocity and acceleration. 		

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<p>Feb.- March (20 days)</p>	<ul style="list-style-type: none"> • Compute derivative of $\frac{dx}{d\theta}$, $\frac{dy}{d\theta}$, $\frac{dy}{dx}$ analytically and on the graphing calculator. • Compute the area of a region bounded by polar curves. • Graph polar functions, including circles, roses, limacons and cardioids, by analytic methods and using the polar mode of graphing calculator. <p><u>Sequences and Series</u></p> <ul style="list-style-type: none"> • Recognize and apply examples to specific series including geometric, harmonic, p-series and alternating harmonic. • Apply convergence tests to series with all positive terms including nth term, Integral, Ratio, Root, Limit Comparison and Direct Comparison (IRRLC mnemonic) • Recognize and apply geometric convergence rule $\frac{a}{1-r}$ for $r < 1$. • Recognize and apply p-series convergence. • Analyze alternating series convergence using Leibniz Theorem. • • Analyze and apply alternating series error bound 	<p>Use the "list" feature of the graphing calculator to explore convergence of a sequence of partial sums and relate that limit, if it exists, to the convergence of a series.</p> <p>Reemphasize, since we are nearing the end of the course, how to show work related to justifying the convergence of a series.</p> <p>Students continue to take turns presenting correct solutions to AP style free response questions to the class.</p>	

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<p>March – Apr. (8 days)</p>	<p>series error bound.</p> <ul style="list-style-type: none"> Analyze series for absolute and conditional convergence. Develop and find Taylor and Maclaurin series for given functions and use them to estimate values of x. Analyze the specific Maclaurin series for $\frac{1}{1-x}$, e^x, $\sin x$ and $\cos x$. Perform formal manipulation of Taylor series and shortcuts to computing Taylor series, including substitution, differentiation, antidifferentiation and formation of a new series. Find radius and intervals of convergence. Analyze and calculate error using LaGrange Error Bound and the Remainder Theorem. <p><u>Parametric and Vector Functions</u></p> <ul style="list-style-type: none"> Analyze parametric motion on graphing calculator, including window analysis in parametric mode. Calculate and analyze of velocity and acceleration, including contrasting these to rectilinear motion. Calculate analytically and verify on the graphing calculator <p>$\frac{dx}{dt}, \frac{dy}{dt}, \frac{d^2x}{dt^2}, \frac{d^2y}{dt^2}$</p>	<p>Have students build the Maclaurin series and identify patterns for rules for e^x, $\sin x$ and $\cos x$. Use this opportunity to rigorously review functions that are even, odd or neither.</p> <p>Have students explore polynomial approximations of transcendental functions such as $\ln(x)$, e^x and $\sin x$ and, using the approximations, estimate function values for specific values of x.</p> <p>Prove $e^{i\pi} - 1 = 0$</p>	

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<p>April (20 days)</p>	<p> $\frac{dx}{dt}, \frac{dy}{dt}, \frac{d^2x}{dt^2}, \frac{d^2y}{dt^2}, \frac{dy}{dx}$ </p> <ul style="list-style-type: none"> Develop the formula and analyze the concept of speed in parametric motion. Develop the formula and analyze the concept of distance traveled along a parametrically defined curve and relate it to length of arc formula. Calculate the angle between vectors (“direction”). Calculate a scalar product. Use vectors to describe motion in a plane including velocity, speed, acceleration and direction of motion. Calculate position, velocity, speed, acceleration and direction algebraically and using the graphing calculator. <p><u>AP Exam preparation</u></p> <ul style="list-style-type: none"> Take and score various practice exams. Review in detail appropriate notations for exam. Emphasize written expression of ideas. 	<p>Have students work together on problems contrasting displacement versus total distance travelled and be sure students use correct vocabulary when helping each other and when explaining strategies in small groups and to the class.</p> <p>It is better to spend a few days focusing on multiple choice and then a few days on free response before moving towards entire practice exams. Have the students take one of the beginning exams in groups to build confidence.</p>	

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<p>May – June (20 days)</p>	<p><u>Various projects and activities</u></p> <ul style="list-style-type: none"> • Group work on logic and problem solving. • In depth analysis of AP Free Response exam questions from current AP, including Form B. • Independent projects. • Internet research on topics not in curriculum, including Work, Force, and Surface Area. 	<p>Have students present solutions to free response questions, focusing on using precise calculus notation and vocabulary. Encourage students to practice presentations at home before presenting to the group, in order to foster smooth, clear and concise presentations.</p> <p>Logic project involving Pirates is always a productive one. Have students reason out their answers and then spend time preparing how to present their justifications.</p> <p>Having students present the AP Free Response solutions is a good way to reinforce their skills at verbal communication of mathematics.</p>	